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Theory of Classical Gaussian Observer

By Henrik Stenlund
Visilab Signal Technologies Oy

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THEORYOFCLASSICALGAUSSIANOBSERVER

Theory of Classical Gaussian Observer

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I. Introduction

a) Background

The motivation for this paper has been the need to define the general classical physical observation in a satisfactory way. The system consists of a Gaussian measuring instrument (observer) and a target system (target) with a Gaussian distribution. This point of view seems to be overlooked and considered trivial in spite of its importance. The literature around this subject seems to be little. The author dares to complete this issue and put the tile on its place.

The corresponding quantum mechanical aspects have been treated in a great number of articles but even that problem has not found a final solution. These two topics must have some correspondence and common points. Lamb [1], Reece [2] and Zeh [3] are most notable of the recent studies with a well covered list of references therein. In Wheeler [4] is an excellent collection of all important articles on the subject up to 1983. The treatments do not seem to have a consistent handling of observation of probability distributions with Gaussian profiles and the classical point of view has no weight.

In the following is presented a theory of observation of classical physical quantities by using a Gaussian model and based on elements of probability. One is talking interchangeably of measurement and observation. The subject of the measuring system affecting the target system's behavior is not treated since that is mainly a phenomenon of the quantum world and outside of the topic of this paper.

b) General

The general conception in earlier, a bit outdated, articles is that a human is active in the observation process and one has to take into account his brain functioning and other biological processes, like eye sight. This misconception is completely outside the topic since observations can be made by automatic measuring systems, robots and satellites without any human intervention. The incorrect view exists in the 1930's to 1950's in many articles attempting to connect the classical or macroscopic world and quantum mechanics. In the following the human aspect is completely ignored and this is treated as a pure observation irrespective of specific observer details.

Ref

4. J. A. Wheeler, W. H. Zurek, *Quantum Theory and Measurement*, Princeton University Press 1983.

Author: *Visilab Signal Technologies Oy, Finland. e-mail: henrik.stenlund@visilab.fi*

i. *The Process of Observation*

Focusing more closely into observing the value of some physical quantity in a target system, one will soon realize that it is more complicated than advertised. For instance, in spite of the apparent simplicity from the point of view of a physicist, image analysis and pattern recognition in industrial processes are seldom accurate. They contain lots of distorting factors destroying any ideal model [5]. As further examples, to measure visible spectral contents from a galaxy or the fluorescent radiation coming from a single molecule implanted in a crystal, the process becomes very complicated. The actual measuring process usually goes as follows, with one or more aspects dominating the others.

- Locate the target system to be measured, in the spatial dimensions. The need to scan some volume or coordinate range of the space to locate the target system is recognized.
- Identify the target since there may be others similar in the vicinity, within the volume. Some sort of pattern matching is required to ascertain which object one would be dealing with.
- Make the actual measurement to the accuracy allowed by the instruments, of the variables intended. The measurement process itself is usually complicated since there are no perfect instruments for measuring any physical quantity. Many types of noise contributions must be eliminated with run-time filtering and post-processing.
- The process will require some time forcing the time to become one of the coordinates. Also very often the target has an interesting temporal dependence (event) requiring simultaneity of the measurement and the event to succeed. The measuring time spent consists of time windowing for analysis, sampling or acceptance time, phase-locking time, sensor rise times etc, depending on the system in question. Claiming that some measurement is an infinitely short delta-function type event is totally false. Time is in the same category as any other measurable quantity in the system.
- Interpret the measurement results correctly. This is self-evident but is not always trivial.
- Repeat the measurements in a completely different way creating results independent of the first ones, if any doubt appears of their validity.

It is now obvious that one would be interested in measuring simultaneously the position and some observable and time. To simplify the initial analysis, in the following a one-dimensional model is set up for making simple measurements and that model is used as a basis for generalization to three dimensions and to adopting an arbitrary quantity for measurement. Quantum mechanical and relativistic phenomena at all stages are ignored. That is done in spite of knowing that quantum physics is generally considered more profound than classical physics. This starting point is justified until the quantum mechanical measurement problem has a complete solution, possibly extendable to a macroscopic system and classical variables.

II. Physics of Observation

a) *The Observer*

The observer function $g(x, t)$ describes the ability of the observing instrument to measure a specific observable x and is blurred around the peaking value at the origin, no matter how accurate instruments there are. They always contain noise and drift of different types in varying frequency bands, generated by many physical phenomena. In addition, other unwanted signals are affecting the end result. Traditionally, an instrument does have a Gaussian distribution

Ref

5. Nelson Bridwell, *Maximizing product quality and yield using vision systems*, Vision Systems Design, Feb 2016.

in its observables. The uncertainty spreads to the time coordinate too since no system is able to make measurements in zero time. Often delta-function like measurements have been assumed and the preceding fact ignored. The physical division between the target and observer is reasonable to be made immediately outside the target since the target is what is required to be measured, not anything that affects the measurements outside of it. The external phenomena do not belong to the target variable and must be isolated.

Things get more complex when smaller targets are studied and approach the microscopic and atomic world. The variables measured can be practically any physical quantities like position, momentum, radiative content with extensive analysis etc. but actual quantum mechanical phenomena are left outside the scope. Position is considered a fundamental variable in many systems; therefore it is picked up for our examples.

For observing dynamic phenomena, like the velocity of a target, the observer is acting in its own inertial frame of coordinates. It should not be subject to significantly interfering interactions with the rest of the world. The observer is not part of the laws of physics in the events of the target. The observer only obeys its own laws mostly associated with the observation itself. Things change gradually when the target size becomes of microscopic order. Observer's influence on the target will become more perceivable if it needs to send some excitation to the target of atomic magnitude.

The observation needs to be complicated with the following common realities. The target and observer may be in accelerating curvilinear relative motion. Also the medium (e.g. gases) carrying the primary measurement signals (usually electromagnetic radiation or acoustic waves) may be in motion relative to the target. The medium's volume may consist of complex flows and rotors and be most inhomogenous in consistency. The medium itself may generate disturbing radiation without external excitation or be selectively absorbing. These facts will affect measurements directly in many practical cases.

There is no perfect observer nor instrument and never will be. This is illustrated in Fig. 1 as a placeholder for an ideal instrument covered with a blurring wall separating it from the target.

b) The Target System

The target system is here referring to an object whose particular physical quantity one intends to measure. The measurement can be focusing on one quantity only but can cover a great number of them as well, to be measured either simultaneously or independently. As an important example is taken the coordinate of the target in one dimension. It is common to treat the target position as an ideal point or its outline dimensions like a hard-core stable object. In reality, the target's variable will have a blurred distribution $y(x, t)$ in the coordinate due to various reasons. The coordinate of a classical object is not so accurate as one might expect (specified as the center of gravity). This thought was suggested already by Heisenberg [6] and Scrödinger [7]. A recent discussion of this was by Mehdipour [8] pointing out the possibility of having Gaussian distributions.

The object may have a varying velocity due to a number of external forces (e.g. Brownian movement), thermal expansions in its volume, extra atomic layers on top of it (e.g. a monolayer of water molecules). It may be rotating at a fast rate or have an inaccurate volume boundary and a complicated varying three-dimensional structure rendering difficult the exact specification of its position. It may be losing or gaining energy for some unexpected reason and numerous other interactions may affect. The exact location of the center of gravity is not stable in a macroscopic object and surely has a distribution. The smaller in the size of the target particle one goes, the relatively more blurred it becomes due to interactions with the surroundings. A good example is a small molecule whose atoms are vibrating and it is impossible to exactly set its center

Ref

8. S. Hamid Mehdipour, *Entropic force law in the presence of noncommutative inspired spacetime for solar system scale*, arXiv:1605.03409v1 [physics.gen-ph], 2 May 2016.

of gravity, even in a crystal lattice. Similar change, while going to the small, may happen to all other physical variables, some are more vulnerable than the others. Obviously, some of the facts listed may as well be overlapping with the features of the imperfect observer itself. One cannot always draw a clear borderline between the two sources of uncertainty.

A great example of an observable which always has a significant uncertainty is the temperature of an object. It has both a distribution inside macroscopic objects and temporal fluctuations and may be subject to endothermic or exothermic processes. One would need a precise way of defining the target temperature, irrespective of the apparent triviality. The measurement itself would be based on infrared radiation from the surface or on some indirect method, like a Platinum resistor mounted inside. They are both far from being perfect in absolute precision although they can offer a fair repeatability and resolution with a relatively low noise. This fact is immediately reflected on the distribution of the variable itself.

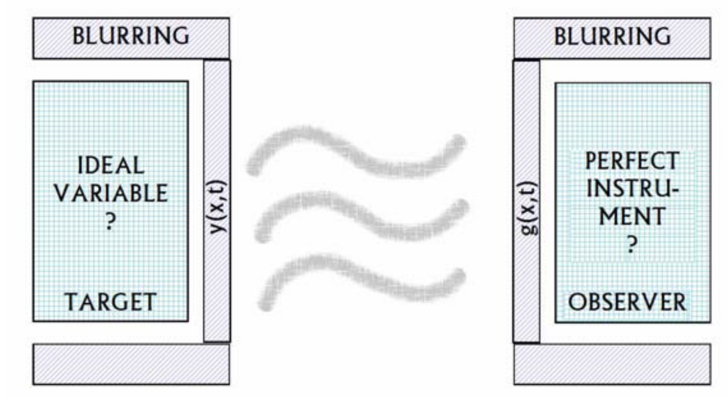

Figure 1: Observation with blurring

i. *Uncertainty Relations*

One could argue that physical quantities themselves are ideal to measure and have no distribution but this has not yet been proven. On the contrary, not even on the classical level can be stated that all, if any, variables would be ideal. When the atomic scale is approached, the particles are acknowledged to have distributions of probability instead of precise ideal values. In the microscopic world the Heisenberg and other uncertainty relations give estimates and conditions for variables' limiting accuracy. For instance, infrared radiation at $\lambda = 10\ \mu m$ whose frequency one needs to measure from one or a few photons. One insists on having a fair accuracy of 15 digits. The Heisenberg uncertainty relation suggests an uncertainty in time of the order of a few seconds, while using a perfect measuring instrument. It would not be reasonable to suggest making a zero-time delta-function type measurement of this observable. While measuring spectra of atomic emission having broad peaks, one can easily have a situation where the target is restricting the measurement's accuracy and cannot be made any better even with a perfect instrument, if there would be any. Even the spectral line width of a freely radiating cold atom is not zero. It can easily be calculated.

All this unavoidably brings to mind that there is some sort of internal uncertainty associated with each variable, including time, affecting the measurements but being independent of the observer. Traditionally, it is expected that things are relatively more accurate with a growing target mass. That is partly true but other phenomena start to creep in. There is no such thing as an ideal variable. Refer to the Fig. 1. There is a placeholder for it behind a blurring wall.

Notes

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c) Constructing the Observation

i. Distributions for the Target and Observer

The conclusion from the facts in the preceding paragraphs is that probability distributions for each observable exist, including the time, and for the observer. The resulting observation becomes a probability distribution. No quantum mechanical effects as such are taken into account. In astronomical measurements one would be limited by restrictions caused by the event horizon due to extremely long distances and possibly high velocities.

Notes

ii. Distribution for Observation

The fact that there is only one kind of target in the volume one is interested in, is assumed. In the following one is concentrating on measuring the coordinate of the target. Also it is assumed that the range of interest for the spatial coordinate will be $(-L, L)$ and for the temporal coordinate $(-T, T)$. The observation can be performed in one dimension or variable at a time as a process of summing the contribution of infinitesimal parts throughout the volume. Simultaneously one runs through with the observer function and progress from positive to negative direction. The infinitesimal probabilities for the simultaneous measurements in x' and t' are $\Delta p'_x$ and $\Delta p'_t$ respectively with corresponding infinitesimal widths $\Delta x'$ and $\Delta t'$

$$\Delta p'_x \Delta p'_t = \Delta x' \Delta t' y(x', t') g(x - x', t - t') \tag{1}$$

Summing the infinitesimal probabilities along x' and t' will lead to a double integral forming the observation at (x, t)

$$z(x, t) = \int_{-L}^{L} dx' \int_{-T}^{T} dt' y(x', t') g(x - x', t - t') \tag{2}$$

The $g(x, t)$ function is normalized properly for both integrations. $g(x, t)$ will be independent on the details of the target function $y(x, t)$ and determined by the measuring instrument and by the details of the measurement process.

iii. Three-Dimensional Distribution for Observation

In three dimensions there is a straightforward extension to

$$z(\vec{r}, t) = \int_V d\vec{r'} \int_{-T}^{T} dt' y(\vec{r'}, t') g(\vec{r} - \vec{r'}, t - t') \tag{3}$$

The functions z, y are scalar functions of vectors but can be vector functions of vectors in vectorized cases and the multiplication specified properly.

III. The Gaussian Model

a) One-dimensional Model

In the following a simple Gaussian peaking observation function and a basic single-variable target having the same nature are prepared. The distribution functions can accept other than Gaussian forms but will not likely cause significant qualitative changes in equations, except add some mathematical inconvenience. One requirement is that the distribution approaches zero quickly after a few half-widths away from the peak, with both functions. The use of a Gaussian is well established in statistical processes and it brings to the analysis certain easiness in integration without having to fall back on piecewise integration or complicated approximation methods.

The observer's and target's distribution functions can be multipeaking, according to the system's specific requirements. The systems may consist, for instance, of multiple states and the exact state is not predictable. Thus a multipeaking Gaussian may be justified for the target which can be approximated well with exponential functions allowing easy integrability.

i. *The Observer*

The observer function is expected to behave as a Gaussian around the origin in both coordinates (x,t) as

$$g(x,t) = \frac{1}{MN} e^{-\kappa x^2 - \xi t^2} \tag{4}$$

x and t are coordinates in the range within which the target lies and which are an active part of the observation process. Here M, N are normalization constants, evaluated with a constant target distribution y. Normalization will give a unity observation if the $y(x,t)$ is unity, indicating that the target is within the volume but one cannot say where and when. The peak width in x-coordinate of this distribution is $1/\sqrt{\kappa}$ and the temporal width is $1/\sqrt{\xi}$.

ii. *The Target*

The target has a Gaussian distribution of probability of the position x and time t

$$y(x,t) = e^{-\beta(x-\hat{x})^2 - \eta(t-\hat{t})^2} \tag{5}$$

Here $\hat{x}$ is the position variable's expectation value which is the ideal variable having an infinite accuracy if ever possible. Correspondingly, $\hat{t}$ indicates the ideal (expectation) value for the time when the target can be located at the point $\hat{x}$. See the Figure 2. below. The resulting observation of the Gaussian particle in one dimension will be the following

$$z(x,t) = \frac{1}{MN} \int_{-L}^{L} dx' \int_{-T}^{T} dt' e^{-\beta(x'-\hat{x})^2 - \eta(t'-\hat{t})^2} e^{-\kappa(x-x')^2 - \xi(t-t')^2} \tag{6}$$

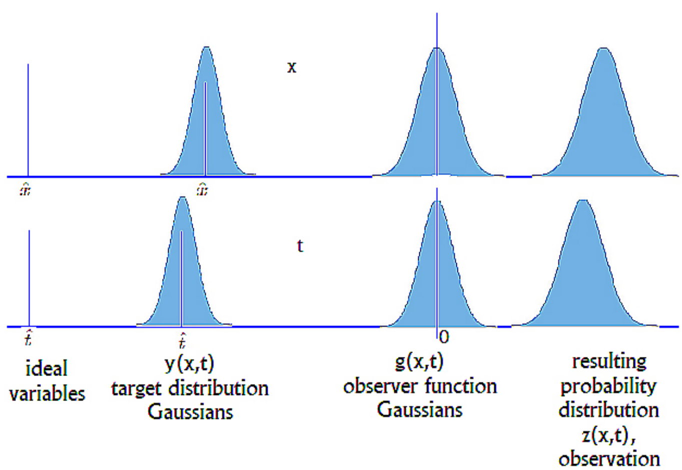

Figure 2: A crude sketch of the observation process with Gaussian distributions. To the left are the Dirac delta function distributions of the ideal variables and while proceeding to the right through each stage the distributions become wider

iii. *Infinite Ranges*

As agreed above, the target distribution and the observer functions fall rapidly to zero outside the peak and therefore one can let the limits of integration L and T to go to infinity, since it is expected not to make observations near the boundaries.

Notes

$$z(x,t) = \frac{1}{MN}\int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dt' e^{-\beta(x'-\hat{x})^2-\eta(t'-\hat{t})^2} e^{-\kappa(x-x')^2-\xi(t-t')^2} \tag{7}$$

In this model the normalizations for x'- and t'-integrations will become

$$\frac{1}{M} = \sqrt{\frac{\kappa}{\pi}} \tag{8}$$

$$\frac{1}{N} = \sqrt{\frac{\xi}{\pi}} \tag{9}$$

Notes

Thus one gets after integration

$$z(x,t) = \sqrt{\frac{\kappa\xi}{(\kappa+\beta)(\eta+\xi)}} e^{-\frac{\kappa\beta(x-\hat{x})^2}{\kappa+\beta} - \frac{\xi\eta(t-\hat{t})^2}{\eta+\xi}} \tag{10}$$

In the following is studied limiting cases for this expression.

iv. *Accurate Observer Limit*

If the observer's Gaussian is narrow compared to the target's Gaussian ($\beta << \kappa, \eta << \xi$), one expects to get rather accurate results. The observation becomes

$$z(x,t) \approx e^{-\beta(x-\hat{x})^2-\eta(t-\hat{t})^2} \tag{11}$$

which is what traditionally is expected of this measurement. The instrument's capability is not restrictive in this case.

v. *Inaccurate Observer Limit*

In case the observer's Gaussian is broad compared to the target's Gaussian ($\beta >> \kappa, \eta >> \xi$), one gets

$$z(x,t) \approx \sqrt{\frac{\kappa\xi}{\beta\eta}} e^{-\kappa(x-\hat{x})^2-\xi(t-\hat{t})^2} \tag{12}$$

The observation distribution has flattened wider compared to the more accurate case above.

vi. *Dirac Delta Function*

It is interesting to note that our observer function

$$g(x,t) = \frac{\sqrt{\xi\kappa}}{\pi} e^{-\kappa x^2-\xi t^2} \tag{13}$$

is precisely the definition of the Dirac delta function in the limit of growing κ and ξ, treated separately.

$$\lim_{\kappa\to\infty,\ \xi\to\infty} g(x-x', t-t') \to \delta(x-x')\delta(t-t') \tag{14}$$

This gives some justification for the traditional assumption of infinitely fast and accurate measurements, in the limit of extremely sharp Gaussian of the observer, both in time and spatial coordinates. The Dirac delta function will let the $y(x,t)$ to emerge from the integrals (7) offering it as the result of measurement.

vii. *Accurate Target Limit*

If the target's Gaussian becomes narrow to the limit of Dirac delta function, it will push out the $g(x,t)$ from the double integral (10)

$$y(x,t) = \delta(x-\hat{x})\delta(t-\hat{t}) \tag{15}$$

$$z(x,t) = g(x,t) \tag{16}$$

The result will be the observer's distribution. The Gaussian $y(x,t)$ does not become a Dirac delta function automatically just by narrowing its Gaussian width but must in that case be the distribution of the target as with the observer function, with a multiplier of $\sqrt{\beta}$ and/or $\sqrt{\eta}$.

Notes

b) Adding a Simultaneous Variable for Measurement

Suppose there is a physical quantity u and the target distribution is the following

$$y(x,t) = e^{-\beta(x-\hat{x})^2-\eta(t-\hat{t})^2-\gamma(u-\hat{u})^2-\rho(t-\hat{T})^2} \tag{17}$$

One has added a new time $\hat{T}$ indicating the moment of proper measurement of the variable u having a specific ideal value $\hat{u}$. To test if the added time Gaussian has some meaning one calculates the observation with the observer function

$$g(x,t) = \frac{1}{MNK}e^{-\kappa x^2-\xi t^2-\alpha u^2} \tag{18}$$

and perform the integration to get

$$z(x,t) = \sqrt{\frac{\kappa\xi\alpha}{(\kappa+\beta)(\eta+\xi+\rho)(\alpha+\gamma)}}e^{-\frac{\kappa\beta(x-\hat{x})^2}{\kappa+\beta}-\frac{\alpha\gamma(u-\hat{u})^2}{\alpha+\gamma}-\frac{\xi\eta(t-\hat{t})^2+\xi\rho(t-\hat{T})^2+\rho\eta(\hat{t}-\hat{T})^2}{\eta+\xi+\rho}} \tag{19}$$

One can immediately see that this expression is nonzero only if $\hat{t} \approx \hat{T}$. It is equivalent to having exactly the same measuring time for all simultaneous measurements. The contribution of simultaneous observation of the variable u is with the common temporal term shown

$$z(u,t) = \sqrt{\frac{\alpha}{\alpha+\gamma}}e^{-\frac{\alpha\gamma(u-\hat{u})^2}{\alpha+\gamma}-\frac{\xi\eta(t-\hat{t})^2}{\eta+\xi}} \tag{20}$$

This is peaking nicely at $\hat{u}$ as it is supposed to. The width of the observational distribution is affected by α. If an added measurement is independent of the original measurement performed, the end result of the observation is additive. For simultaneous dependent measurements, it is multiplicative.

IV. Discussion

The classical physical quantities behaving according to the laws of physics is one thing and measuring them is another. The measurement results can approach accurate values if the measuring conditions are favorable and the instruments have suitable properties, i.e. their Gaussian widths are extremely narrow approaching Dirac delta functions in form. However, they are not the same except by chance, since no perfect instruments exist and the target's variable will also have a Gaussian distribution due to its own uncertainties. The results of measuring classical quantities will always have probability distributions based both on uncertainties of the target system and on imperfections in the observer. Ideal variables are good for theories but exist only in the minds of physicists; they are affected by blurring.

One takes into use a Gaussian distribution both for the observer and for the target system's variable to be measured. It will give a model which is closer to reality than hard core type objects and Dirac delta-function type measurements which are ideal and nonexistent. The observation is a Gaussian in many cases.

The main results of this work are equations (2) and (10).

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